Assignment N3

In this assignment you will use the singular value decomposition (svd) of matrices to solve two different problems.

1 Least squares problem with a linear constraint

Write an m-function `x=lsqlc(A,b,C,d,tol)` that solves least squares problems with linear constraints,

$$\begin{array}{ll} \min & \|Ax-b\|. \\ Cx = d & \end{array}$$

If there are several solutions, the function should return the one with the minimal $\|x\|$. Here $\|\cdot\|$ is the 2-norm of a vector, A – an $m \times n$ matrix, b – an $m \times 1$ vector, C – a $p \times n$ matrix, d – a $p \times 1$ vector, *tol* – tolerance. Suggested method:

1. Find the svd of matrix C, $C = USV^T$. Set all the singular values which are less than *tol* to zero. Use the decomposition obtained to analyze the system $Cx = d$ and find its minimal norm solution x_0. If no solution exists, return $x = NaN$. That would mean "no solution to optimization problem exists".

2. If there exists a solution x_0 to $Cx = d$, all other solutions to this system may be written as $x = x_0 + y$, where $Cy = 0$. Let $k = rank(C)$ (only k singular values of C are greater than *tol*, the others were set to zero). Using the svd of C we obtain that $USV^T y = 0$, hence $y = Vz$, where the first k coordinates of vector z are zeros and all other coordinates can be arbitrary: $z = (0, ..., 0, z_{k+1}, ..., z_n)^T$.

3. Since $x = x_0 + Vz$, we have to minimize $\|AVz - \widehat{b}\|$, where $\widehat{b} = b - Ax_0$, over all such vectors z. Note that first k columns of AV are unimportant because first k coordinates of z are zeros. Let G be the matrix consisting of columns $k+1, ..., n$ of AV. Use svd (or pseudoinverse) of G to find the minimal norm solution, $u = (u_1, ..., u_{n-k})^T$, to the least squares problem $\min \|Gu - \widehat{b}\|$. Set $z = (0, ..., 0, u_1, ..., u_{n-k})^T$ and $x = x_0 + Vz$.

You may use Matlab's `svd` function (as well as the `pinv` function if you need it).

2 Data compression for black and white images

Write an m-function `im_compr(image,frmt,compr)` that reads an image file `image` written in format `frmt` using Matlab's function `X=imread(image,frmt)`, transforms the `uint8` array X into a matrix, `X=double(X)`, and then uses svd of X to compress the data in the ratios `compr(1), compr(2), ...` The program output consists of several graphs: the original image, a plot of the singular values of X, the images obtained after the data compression. Use `imagesc(X)` to draw the images and `colormap('gray')` (black and white images). In your report discuss the compression results for two high-resolution images from the NASA Photo Gallery of Asteroids, `gaspra.jpg` and `idasmoon.jpg` (The photos have been obtained from the Galileo spacecraft, the first planetary mission to photograph an asteroid "up-close". Its flyby of Gaspra occurred on 29 October 1991 at a distance of about 16,200 km. The second of the two asteroids which Galileo encountered en route to Jupiter, Ida was discovered to have something different: its own satellite! Galileo's flyby of Ida and its moon Dactyl occurred on 28 August 1993 at a distance of about 2,400 km.)