*Introduction to Numerical Analysis, Fall 2002*

# Assignment N2

- **Aircraft tracking** – In this work you will write an m-program `flight` to track simultaneously a number of airplanes. This can be accomplished in a surprising way using polynomials, splines, and interpolation.

  *DATA*: For each of the $Nt$ time moments, $t = 0, \frac{1}{Nt-1}, \frac{2}{Nt-1}, ..., 1$, $Np$ points in the plane, $Z(t, j),\ j = 1, ..., Np$, are given and represent airplanes on your radar screen.

  *YOUR MISSION*: To draw the trajectory of each airplane in a different color. The main difficulty here is to determine which airplane is which: the blips on the radar screen do not have numbers.

  *METHOD*:
  Consider the plane as the complex plane, so that a point $(x, y)$ is represented as $x + iy$.

  1. For each time $t$ construct the polynomial whose roots are the points $Z(t, j),\ j = 1, ..., Np$, and use Matlab to find the (complex) coefficients of this polynomial, $a_m(t),\ m = 0, 1, ..., Np$.
  2. For each $m$ you have now a sequence of values $a_m(t),\ t = 0, \frac{1}{Nt-1}, \frac{2}{Nt-1}, ..., 1$. Construct, using Matlab's functions
     (a) the cubic spline interpolant to $a_m(t)$.
     (b) the least squares approximation by a polynomial of degree $k$.
  3. For the time moments $t = 0, h, 2h, ..., 1$ calculate approximate values of $a_m(t)$, in case
     (a) by using the cubic spline interpolation,
     (b) the polynomial approximation.
  4. Use Matlab to find roots of the polynomial

     $$R_t(z) = a_0(t) + a_1(t)z + ...$$

     at times $t = 0, h, 2h, ..., 1$. Choose $h$ small enough so that each airplane has moved only a small distance. Your m-function should now match the airplanes and display the trajectories of each airplane in a different color.

  You may use any Matlab's function you need (use "help polyfun" command to learn about the functions that can be helpful).

  Typical values are: $Np = 4 - 6,\ Nt = 25 - 40,\ k = 15 - 20,\ h = 2 \cdot 10^{-4} - 5 \cdot 10^{-4}$.

  Experiment with other values, and with different radar pictures. (You may copy and use our simple m-function `inp_gen` that generates nice "random" trajectories and can produce the input data for your `flight` program.) Sometimes the trajectories may look weird. Can you think of an explanation? Can you propose a generalization of this method for $R^3$?
  Can you suggest a different method?

- **Analysis of FACS histograms** – We ask you to analyze the fluorescence histograms obtained using the Fluorescence-Activated Cell Sorter (FACS) at the Bone Marrow Transplantation Department of Hadassah hospital.

The binary file *data.mat* contains five vectors of the same length, $f1$, $f2$, $f3$, $f4$, and $fm$ (to get them, download the file and use Matlab's command "load data"). Here $f1, ..., f4$ are histograms characterizing the distribution of fluorescence levels in four different populations of cells stained by a fluorochrome (see figure). The vector $fm$ contains a similar histogram for a mixture of these cell populations. Your goal is to estimate the concentrations $c1$, $c2$, $c3$, $c4$ of each of the populations in the mixture by using the least squares method. No program needs to be submitted. In your report describe the model you used (note that $c1 + c2 + c3 + c4 = 1$) and how you calculated the unknown concentrations. Present the concentration values and a graph showing the histogram of the mixture and its fit by the mixture of histograms.

*Figure*: Histograms $f2$ and $fm$. To build these histograms, the possible range of fluorescence levels was divided into 200 intervals. FACS measured fluorescence levels of about twenty thousands of cells from each population and calculated, say, $f2(i)$ as the number of cells from the 2-nd population in the i-th interval divided by the total number of analyzed cells from that population.