Assignment N1

1. Write a Matlab function `y=psi(x,rel_err)` that computes values of

$$\psi(x) = \sum_{k=1}^{\infty} \frac{1}{k(k+x)}$$

for any $x > 0$ with the relative error not exceeding `rel_err`.

The main problem is that too many terms may be required because the series converges slowly. Additionally, if enough terms were to be summed, roundoff would render the computation useless for the desired precision. To accelerate the convergence note that

$$\frac{1}{k(k+1)} = \frac{1}{k} - \frac{1}{k+1}$$

which implies $\psi(1) = 1$. One can then produce a series for $\psi(x) - \psi(1)$ which converges faster than the original series. This process of finding a faster converging series can be repeated again on the second series to produce a third sequence, which converges even more rapidly than the second one (Hint: try to find the sum of the second series for $x = 2$.)

The m-function `psi(x,rel_err)` should first determine, for a given x, how many terms of this third series are needed to reach the given accuracy; then it makes the summation and computes $\psi(x)$. Your written report should contain a brief description of the mathematical algorithm employed and the table of 21 values of ψ at the points $x = 0,\ 0.1,\ 0.2, ..., 2.0$ calculated with the relative error not more than 10^{-10}.

The following inequality is helpful in determining how many terms are required in summing the series above,

$$\sum_{k=n+1}^{\infty} \frac{1}{k^m} < \int_n^{\infty} \frac{dt}{t^m}, \quad \text{for } m > 1.$$

To print the table with the necessary number of decimal places use Matlab's function `fprintf`.

2. Write an m-function `secant` that solves nonlinear equations by the secant method. Use this program to solve the equation $\psi(x) = 0.7$ (where ψ is the function defined above) with the absolute error not exceeding 10^{-6}. Explain in your written report why do you think the needed accuracy has been obtained.

3. Use Matlab's program `fzero` to plot graphs of functions given implicitly by equations $F(x, y) = 0$. Your program (`im_plot`) should call `fzero` for different values of one of the two variables to determine the zeros of F as a function of the other variable and plot the points so found. Plot graphs of the following functions, $y(y^2 - 1)(y - 2) = x(x - 1)(x - 2)$ and $y^4(x + 2) + y^2(2x^2 - 16x + 2x^3) + x^5 = 0$. Attach these graphs to your written report.

4. Write an m-function `newton_2` to solve systems of two nonlinear equations

$$\begin{cases} F_1(x, y) = 0 \\ F_2(x, y) = 0 \end{cases}$$

by the two-dimensional Newton method. As an example of the use of this program find all solutions of the system

$$\begin{cases} x^5 - x^3 - y^3 + xy + 0.1 = 0 \\ x^2 + y^2 - 1 + 0.2\sin(8xy) = 0 \end{cases}$$

To do this call `im_plot` to plot both functions, and from this plot find first the approximate graphical solutions of the system (these solutions are the intersection points of two graphs and you can use Matlab's function `ginput` to obtain their coordinates interactively). The graphical solutions should serve as good initial approximations for your program `newton_2`. The written report should contain a statement of the problem, a brief description of the mathematical algorithm, and the solutions obtained.

Attention: You are given specifications for each of the programs to be submitted. We expect you to copy and use these spec files.